Note: In all written assignments, please show as much of your work as you can. Even if you get a wrong answer, you can get partial credit if you show your work. If you make a mistake, it will also help the grader show you where you made a mistake.

Problem 1:
Write a program (in pseudo code) that generates 100000 random variables that are (discretely) uniform distributed as follows:

#1 5%
#2 10%
#3 5%
#4 12%
#5 25%
#6 22%
#7 21%

Problem 2:
If \( x_0=5 \) and \( x_n=3x_{n-1} \mod 150 \)
find \( x_1, ..., x_{10} \).

Problem 3:
Use simulation to approximate the following integrals.

a. \( \int_0^5 x^2 \, dx \), also calculate the exact solution.
b. \( \int_0^1 (1 - x^2)^{3/2} \, dx \)
c. \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \)

Problem 4:
Provide the inverse transform algorithm for generating a binomial \((n,p)\) random variable. Recall that
\[
P\{X = i\} = \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} \quad i = 0, 1, ..., n
\]
and make use of the recursive identity
\[
P\{X = i + 1\} = \frac{n - 1}{i + 1} \frac{p}{1 - p} P\{X = i\}.
\]

Problem 5:
Use the inverse transform method to generate a random variable having distribution function
\[
F(x) = \frac{x^2 + x}{2}, \quad 0 \leq x \leq 1.
\]
Problem 6:
We are interested in simulating a single-server queueing system to determine such quantities as the average time a customer spends in the system and the average time past $T$ that the last customer departs.

We use the following variables:
- **Time Variable** $t$
- **Counter Variables** $N_A$: the number of arrivals (by time $t$)
  $N_D$: the number of departures (by time $t$)
- **System State Variable** $n$: the number of customers in the system (at time $t$)

The simulation will begin by the initialization of the variables and the event times $t_A$ and $t_B$.

**Initialize**
Set $t=N_A=N_D=0$.
Set $SS=0$.
Generate $T_0$, and set $t_A=T_0$, $t_D=\infty$.

Fill in the missing lines for cases 1-4!

**Case 1:** $t_A \leq t_D$, $t_A \leq T$
Reset: $t=____$
Reset: $N_A=____$
Reset: $n=____$
Generate $T_0$ and reset $t_A=____$
If $n=1$, generate $Y$ and reset $t_D=____$
Collect output $A(N_A)=____$

**Case 2:** $____$, $t_D \leq T$
Reset: $t=____$
Reset: $n=____$
Reset: $N_D=____$
If $n=0$, reset $t_D=____$; otherwise, generate $Y$ and reset $t_D=____$
Collect output data $D(N_D)=____$

**Case 3:** $\min(t_D,t_A)>T$, $n____$
Reset: $t=t_D$
Reset: $n=____$
Reset: $N_D=____$
If $n>0$, generate $Y$ and reset $t_D=____$
Collect output data $D(N_D)=____$

**Case 4:** $\min(t_D,t_A)>T$, $n____$
Collect output data $T_p=max(t-T,0)$.